Kurs Datenbankgrundlagen und Modellierung

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Sommersemester 2023

22.5.2023
Vorlesung 5: SQL Wrap Up &
Functional Dependencies

Agenda

- 1.) RECAP
 - outer joins
 - subqueries
- 2.) NOT IN / NOT EXISTS
- 3.) UNION (ALL) / EXCEPT (ALL)
- 4.) ANY / ALL

- 5.) Functional Dependencies
- 6.) Armstrong Axioms

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```
17.4. Vorlesung 1 — Intro
24.4. V2 — ER, SQL
1.5. keine Vorlesung
4.5. Fragestunden finden statt
8.5. V3 — SQL
10.5. Ü1
11.5. Fragestunden
15.5. V4 — SQL
17.5. Ü2
22.5. V5 — SQL
24.5. Ü3
25.5. Fragestunden
29.5. V6 — funct. dependencies
31.5. Ü4
1.6. Fragestunden
5.6. V7 — normal forms
7.6. Ü5
```

8.6. Fragestunden

```
12.6. V8 — modelling Intro
14.6. Ü6
15.6. Fragestunden
19.6. V9 — class diagrams
21.6. Ü7
22.6. Fragestunden
26.6. V10 — state charts
28.6. Ü8
29.6. Fragestunden
3.7. V11 — sequence diagrams
5.7. Ü9
6.7. Fragestunden
10.7. V12 — Klausurvorbereitung
```

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Wintersemester 2023/24

Semesterdauer	01.10.2023 - 31.03.2024
Orientierungswochen (inklusive Deutsch Intensivkurs) - optional	25.09. – 13.10.2023
Vorlesungszeit	16.10.2023-02.02.2024
Prüfungszeitraum	S.U.
Veranstaltungsfreie Tage:	
Tag der Deutschen Einheit Reformationstag	03.10.2023 31.10.2023
Weihnachtsferien	23.12.2023 - 05.01.2024



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Sommersemester 2023

Semesterdauer	01.04.2023 - 30.09.2023
Orientierungswochen	20.03.2023 - 06.04.2023
Vorlesungszeit	11.04.2023 - 14.07.2023
Prüfungszeitraum	s.u.
Veranstaltungsfreie Tage:	
Karfreitag Ostern	07.04.2023 09./10.04.2023

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General Form of an SQL Query:

```
FROM list of tables
WHERE condition over attributes
GROUP BY list of attributes
HAVING condition over aggregates
ORDER BY list of attributes
LIMIT number
```

aggregate functions:

```
COUNT VARIANCE
SUM STDDEV
AVG BIT_OR
MAX BIT_AND
MIN
```

1. Outer Joins

- introduce NULLs, when there is no "join partner"
- LEFT / RIGHT / FULL outer joins

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- introduce NULLs, when there is no "join partner"
- LEFT / RIGHT / FULL outer joins

СТ	name	ing
	•	milk kahlua vodka
IG -	ing	alc -+
	vodka	l 40
	milk	1 0
	kahlua	20
	lime juice	I 0
	club soda	1 0
	(5 rows)	

From last year's (2022) exam paper:

Teil 1.e. (1.0 Punkte)

```
SELECT name, COUNT(alc) AS cnt FROM

( SELECT name, alc FROM

CT NATURAL JOIN IG WHERE alc>0 )

NATURAL RIGHT JOIN

( SELECT DISTINCT name FROM CT )

GROUP BY name;
```

What does this query compute?

1. Outer Joins

- introduce NULLs, when there is no "join partner"
- LEFT / RIGHT / FULL outer joins

CT	name ing
	White R. vodka White R. milk White R. kahlua Black R. vodka Black R. kahlua Nojito lime juice Nojito club soda (7 rows)
IG	ing alc vodka 40 milk 0 kahlua 20 lime juice 0 club soda 0
	(5 rows)

From last year's (2022) exam paper:

```
Teil 1.e. (1.0 Punkte)

SELECT name, COUNT(alc) AS cnt FROM

( SELECT name, alc FROM

CT NATURAL JOIN IG WHERE alc>0 )

NATURAL RIGHT JOIN

( SELECT DISTINCT name FROM CT )

GROUP BY name;
```

What happens now?

Subqueries

Anywhere in a query where

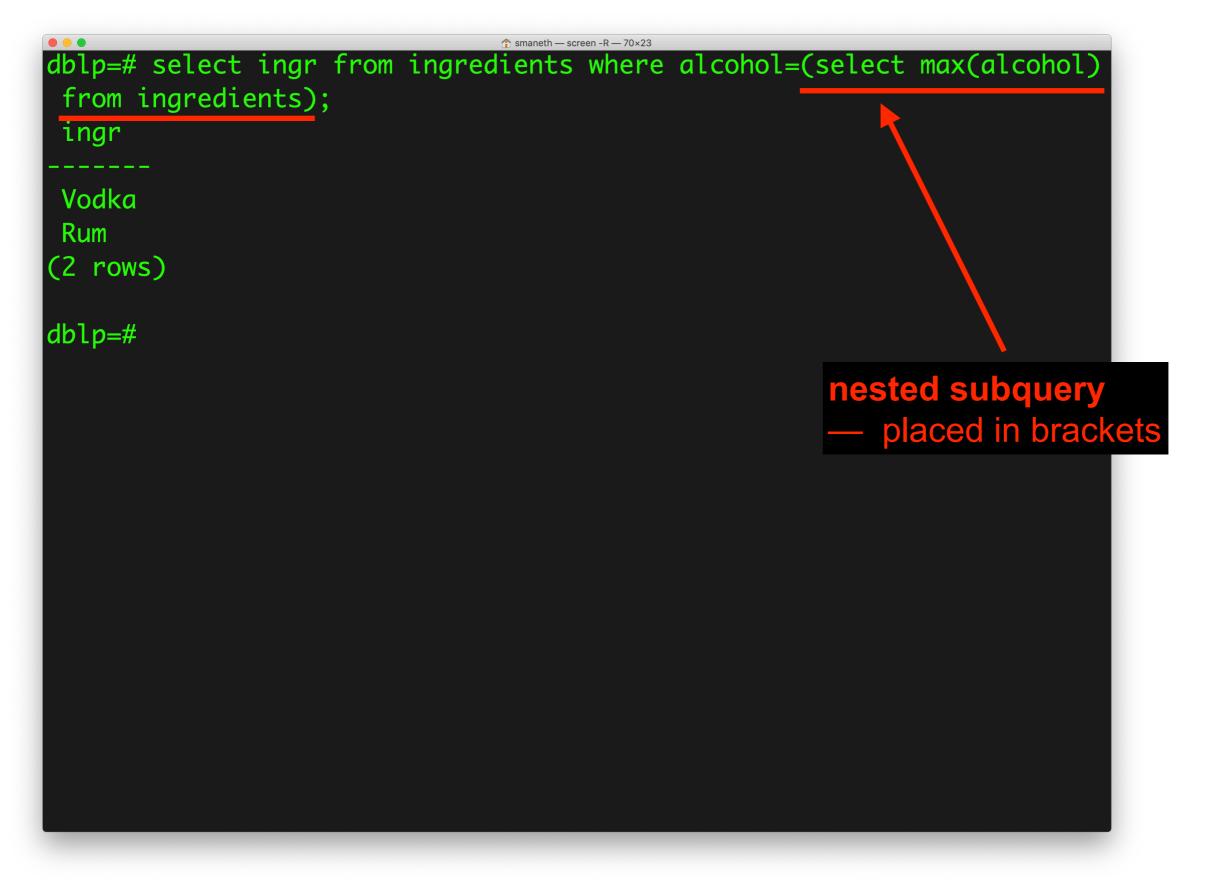
- a table name may appear, we may also place a nested query (a "subquery")
- a value may appear, we may place a query that returns one value

Return ingredients with the maximum alcohol value:

```
↑ smaneth — screen -R — 70×23
dblp=# select * from ingredients;
 ingr | alcohol
 Kahlua | 20.0
           0.0
 Milk
 Vodka |
             40.0
 Rum
             40.0
(4 rows)
dblp=# select max(alcohol) from ingredients;
max
40.0
(1 row)
dblp=# select ingr from ingredients where alcohol=40;
ingr
 Vodka
 Rum
(2 rows)
dblp=#
```

Return ingredients with the maximum alcohol value:

```
nameth — screen -R — 70×23
dblp=# select * from ingredients;
       | alcohol
  ingr
 Kahlua I
             20.0
 Milk
             0.0
 Vodka
             40.0
 Rum
             40.0
(4 rows)
dblp=# select max(alcohol) from ingredients
                                                     Much more elegant:
 max
                                                     nest this query here!
40.0
(1 row)
dblp=# select ingr from ingredients where alcohol=40;
ingr
 Vodka
 Rum
(2 rows)
dblp=#
```



```
dblp=# select ingr from ingredients where alcohol=(select max(alcohol)
 from ingredients);
 ingr
 Vodka
 Rum
(2 rows)
dblp=# select ingr from ingredients where alcohol=(select alcohol from
 ingredients);
ERROR: more than one row returned by a subquery used as an expression
dblp=#
```

Subqueries

Anywhere in a query where

— a table name may appear, we may place a nested query (a "subquery")

Nested queries (that return more than one value) are best demonstrated in conjunction with the **NOT IN / IN** keywords

Non-Monotonic Behavior

SQL queries that use only the constructs introduced above are **monotonic** (\nearrow slide 104).

→ If further tuples are inserted to the database, the query result can only grow.

Some real-world queries, however, demand non-monotonic behavior.

- E.g., "Return all non-alcoholic cocktails (i.e., those without any alcoholic ingredient)."
 - → Insertion of a new ConsistsOf tuple could "make" a cocktail alcoholic and thus invalidate a previously correct answer.

Such queries cannot be answered with the SQL subset we saw so far.

Not true for us because using HAVING and aggregates we already expressed non-monotonic queries.

Non-monotonic queries

Consider this query:

Return all Cocktails that do not contain Vodka.

How would you write it in SQL?

CT	name ing
	White R. vodka White R. milk White R. kahlua Black R. vodka Black R. kahlua Nojito lime juice Nojito club soda (7 rows)
IG	ing alc
	vodka 40 milk 0 kahlua 20 lime juice 0 club soda 0 (5 rows)

Consider this query:

Return all Cocktails that do **not** contain Vodka.

CT	name ing	
	White R. vodka White R. milk White R. kahlua Black R. vodka Black R. kahlua Nojito lime juice Nojito club soda (7 rows)	
IG	ing alc	
	vodka 40 milk 0 kahlua 20 lime juice 0 club soda 0 (5 rows)	

select Name from Cocktails
where ingr='Vodka'

returns Cocktails that contain Vodka

Consider this query:

Return all Cocktails that do **not** contain Vodka.

returns Cocktails that
do not contain Vodka



CT	name ing
	White R. vodka White R. milk White R. kahlua Black R. vodka Black R. kahlua Nojito lime juice
	_
	Nojito club soda
	(7 rows)
IG	ing alc
	vodka 40
	milk O
	kahlua 20
	lime juice 0
	club soda 0
	(5 rows)

select distinct Name from Cocktails
where Name NOT IN (
select Name from Cocktails
where ingr='Vodka');

returns Cocktails that contain Vodka

Consider this query:

Return all Cocktails that do **not** contain Vodka.

returns Cocktails that do not contain Vodka



CT	name ing
	White R. vodka White R. milk White R. kahlua Black R. vodka Black R. kahlua Nojito lime juice Nojito club soda (7 rows)
IG	ing alc
	vodka 40 milk 0 kahlua 20 lime juice 0 club soda 0 (5 rows)

select distinct Name from Cocktails
where Name NOT IN (
select Name from Cocktails
where ingr='Vodka');



Explain why this query is non-monotonic.

Indicators for Non-Monotonic Behavior

Indicators for non-monotonic behavior (in natural language):

- "there is no", "does not exist", etc.
 - \rightarrow existential quantification
- "for all", "the minimum/maximum"
 - → universal quantification
 - $\rightarrow \forall r \in R : C(r) \Leftrightarrow \nexists r' \in R : \neg C(r')$

In an equivalent SQL formulation of such queries, this ultimately leads to a test whether a certain query yields a (non-)empty result.

Indicators for Non-Monotonic Behavior

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Do you know existential / universal quantifiers? (first-order logic)

Poll:

In an equivalent SQL formulation of such queries, this ultimately leads to a test whether a certain query yields a (non-)empty result.

Such tests can be expressed with help of the IN (\in) and NOT IN (\notin) keywords in SQL:

```
SELECT c.Name

FROM Cocktails AS c

WHERE CocktailID NOT IN (SELECT co.CocktailID

FROM ConsistsOf AS co,

Ingredients AS i

WHERE i.IngrID = co.IngrID

AND i.Alcohol <> 0)
```

What does this query compute?

Such tests can be expressed with help of the IN (\in) and NOT IN (\notin) keywords in SQL:

```
SELECT c.Name
FROM Cocktails AS c
WHERE CocktailID NOT IN (SELECT co.CocktailID
FROM ConsistsOf AS co,
Ingredients AS i
WHERE i.IngrID = co.IngrID
AND i.Alcohol <> 0)
```

The IN (NOT IN) keyword tests whether an attribute value appears (does not appear) in a set of values computed by another SQL **subquery**.

→ At least conceptually, the subquery is evaluated before the main query starts.

IN vs. Join

Consider again the query for all alcoholic cocktails.



Do the following queries return the same result?

```
SELECT Name
  FROM Cocktails
WHERE CocktailID IN (SELECT DISTINCT CocktailID
                        FROM ConsistsOf AS co,
                             Ingredients AS i
                       WHERE i.IngrID = co.IngrID
                         AND i.Alcohol > 0)
```

```
SELECT DISTINCT c.Name
 FROM Cocktails AS c, ConsistsOf AS co,
       Ingredients AS i
 WHERE c.CocktailID = co.CocktailID
   AND co.IngrID = i.IngrID AND i.Alcohol > 0
```

IN vs. Join

Consider again the query for all alcoholic cocktails.



Do the following queries return the same result?

```
SELECT Name
      FROM Cocktails
     WHERE CocktailID IN (SELECT DISTINCT CocktailID
                             FROM ConsistsOf AS co,
                                   Ingredients AS i
                            WHERE i.IngrID = co.IngrID
        no!
Only if you add DISTINCT
                              AND i.Alcohol > 0)
  to the upper query!
      SELECT DISTINCT c.Name
        FROM Cocktails AS c, ConsistsOf AS co,
             Ingredients AS i
       WHERE c.CocktailID = co.CocktailID
         AND co.IngrID = i.IngrID AND i.Alcohol > 0
```

EXISTS / NOT EXISTS

The construct NOT EXISTS enables the main (or outer) query to check whether the **result of a subquery is empty**. 9

■ In the subquery, tuple variables declared in the FROM clause of the outer query may be referenced.

```
SELECT Name

FROM Cocktails AS c

WHERE NOT EXISTS (SELECT DISTINCT CocktailID

FROM ConsistsOf AS co,

Ingredients AS i

WHERE i.IngrID = co.IngrID

AND co.CocktailID = c.CocktailID

AND i.Alcohol > 0)
```

What does this query compute?

⁹Likewise, EXISTS tests for non-emptiness.

Correlated Subqueries

The reference of an outer tuple makes the subquery **correlated**.

- The subquery is **parameterized** by the outer tuple variable.
- Conceptually, correlated subqueries have to be re-evaluated for every new binding of a tuple to the outer tuple variable.
 - \rightarrow Again, the DBMS is free to choose a more efficient evaluation strategy that returns the same result (\sim "query unnesting")

Correlation can be used with IN/NOT IN, too.

 \rightarrow Typically, this yields complicated query formulations (bad style).

Queries with EXISTS/NOT EXISTS can be non-correlated.

- → The WHERE predicate then becomes **independent** of the outer tuple.
- \rightarrow This is rarely desired and almost always an indication of an **error**.

Correlated Subqueries

Subqueries may reference tuple variables from the **outer query**.

The converse (referencing a tuple variable of the subquery in the outer query) is **not** allowed:

```
SELECT c.Name, i.Alcohol ← wrong!

FROM Cocktails AS c

WHERE EXISTS (SELECT DISTINCT CocktailID

FROM ConsistsOf AS co,

Ingredients AS i

WHERE i.IngrID = co.IngrID

AND co.CocktailID = c.cocktailID

AND i.Alcohol > 0)
```

→ Compare this to variable scoping in block-structured programming languages (C, Java).

EXISTS / NOT EXISTS

- EXISTS/NOT EXISTS only tests for the **existence** of (at least) one row in the subquery result.
- The actual tuple value returned by the query is immaterial to the overall query result.
- It is good style to make this explicit in the subquery phrasing:

```
ightarrow ... EXISTS (SELECT * FROM ...)

ightarrow ... EXISTS (SELECT NULL FROM ...)

ightarrow ... EXISTS (SELECT 42 FROM ...)
```

■ It is legal SQL syntax, though, to specify arbitrarily complex result tuples in the subquery's SELECT clause.

3.) Set and Multi-Set Operations

UNION

The SQL keyword UNION allows to collect results from multiple queries into a single output relation (\sim algebra operator \cup).

```
SELECT Name, Price
FROM Ingredients
WHERE Alcohol > 0
UNION
SELECT Name, Price
FROM Cocktails
```

UNION is strictly needed (no other way in SQL to express such queries).

Typical use case:

→ Specializations of a general concept are stored in separate tables. They can be re-combined using UNION.

SQL Set Operators

- Combined relations must be schema-compatible.
 - But SQL is less strict than relational algebra.
 - Both operands must have the same number of columns; columns of compatible types must be listed in same order. Column names, however, do not matter (need not be identical).
- The other set operators are available in SQL, too:
 - UNION implements ∪
 - EXCEPT implements (MINUS is synonym)
 - INTERSECT implements ∩
- All three operators remove duplicates.
- To keep duplicates: combine with ALL

```
SELECT···FROM···WHERE···
UNION ALL (or: EXCEPT ALL, INTERSECT ALL)
SELECT···FROM···WHERE···
```

Union has Set-Semantics (no duplicates!)

```
> SELECT * FROM T1;
 ____+
 col1 | col2
> SELECT col1 FROM T1 UNION SELECT col2 FROM T1;
+----+
 col1
```

Union has Set-Semantics (no duplicates!)

— same holds for Intersect and for Except

3.) Set and Multi-Set Operations

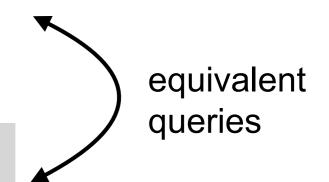
If you want multi-set semantics, use

- Union ALL
- Intersect ALL
- Except ALL

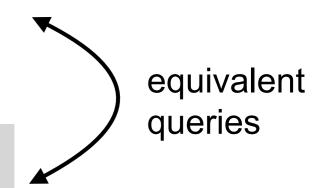
```
> SELECT * FROM T1;
  col1 | col2
> SELECT col1 FROM T1 UNION ALL SELECT col2 FROM T1;
 col1
```

3.) Set and Multi-Set Operations

- 1. "UNION ALL" adds multiplicities
- 2. "INTERSECT ALL" keeps minimum (non-0) number of occ's of an element



do you see yet another way to formulate this query?



union with the *emtpy set*

Find movies that are shorter than some currently playing movie:

```
SELECT M.Title
FROM Movies M
WHERE M.length < (SELECT MAX M1.length)
FROM Movies M1, Schedule S
WHERE M1.title=S.title)
```

Find movies that are shorter than some currently playing movie:

```
SELECT M. Title
FROM Movies M
WHERE M.length < (SELECT MAX M1.length)
                  FROM Movies M1, Schedule S
                  WHERE M1.title=S.title)
or
SELECT M. Title
FROM Movies M
WHERE M.length < ANY SELECT M1.length
                     FROM Movies M1, Schedule S
                     WHERE M1.title=S.title)
```

- <value> <condition> ANY (<query>)
 is true if for some <value1> in the result of <query>,
 <value> <condition> <value1> is true.
- For example,

```
5 < \text{ANY}(\emptyset) is false; 5 < \text{ANY}(\{1,2,3,4\} \text{ is false}; 5 < (\text{ANY})(\{1,2,3,4,5,6\} \text{ is true}.
```

- <value> <condition> ALL (<query>)
 is true if either:
 - <query> evaluates to the empty set, or
 - o for every <value1> in the result of <query>,
 <value> <condition> <value1> is true.

- <value> <condition> ALL (<query>)
 is true if either:
 - <query> evaluates to the empty set, or
- For example,
 - $5 > ALL(\emptyset)$ is true;
 - 5 > (ALL)(1, 2, 3) is true;
 - $5 > ALL(\{1, 2, 3, 4, 5, 6\})$ is false.

fluss	fname	laenge	
	Weser Elbe	452 1091	
durch	fname	sname	laenge
	Weser Elbe	Bremen Dresden Hamburg	45 32 36

fluss	fname	laenge	
	Weser Elbe	452 1091	
durch	fname	sname	laenge
	Weser Elbe	Bremen Dresden Hamburg	45 32 36

What is the **meaning** of this query?

```
X IN S is equivalent to X = ANY S
```

X NOT IN S is equivalent to X != ALL S

4.) Examples with IN

```
SELECT ('a') IN ('a','b');
?column?

t

SELECT ('a','b') IN (('a','b'),('a',NULL));
?column?

t
```

4.) Examples with IN

```
SELECT ('a') IN ('a','b');
?column?
 t
SELECT ('a','b') IN (('a','b'),('a',NULL));
?column?
 t
SELECT ('a', NULL) IN (('a', 'b'), ('a', NULL));
-> ??
```

4.) Examples with IN

```
SELECT ('a') IN ('a','b');
?column?
 t
SELECT ('a','b') IN (('a','b'),('a',NULL));
?column?
 t
SELECT ('a', NULL) IN (('a', 'b'), ('a', NULL));
?column?
 NULL
```

Projection:

Let R be a relation with sch(R) = $(A_1, ..., A_k)$. Let $(i_1, ..., i_m)$ be distinct numbers from $\{1, ..., k\}$. Let $t = (v_1, ..., v_k)$ be a tuple from val(R).

The projection to $(A_{i_1}, ..., A_{i_m})$ is defined as

$$\pi_{A_{i_1},...,A_{i_m}}(t) = (v_{i_1},...,v_{i_m})$$

projection = "select only the given columns"

Let X, Y be *non-empty sets* of attributes of a given table T. The functional dependency (FD) $X \rightarrow Y$ means that for any two tuples t_1, t_2 in val(T):

if
$$\pi_X(t_1) = \pi_X(t_2)$$
 then $\pi_Y(t_1) = \pi_Y(t_2)$.

Let X, Y be *non-empty sets* of attributes of a given table T. The functional dependency (FD) $X \rightarrow Y$ means that for any two tuples t_1, t_2 in val(T):

if
$$\pi_X(t_1) = \pi_X(t_2)$$
 then $\pi_Y(t_1) = \pi_Y(t_2)$.

"if two tuples agree on their X-values, then they also agree on their Y-values"

"the X-values determine the Y-values"

$$\pi_{X,Y}(\mathrm{val}(T))$$
 is a function from X to Y

Let X, Y be non-empty sets of attributes of a given table T. The functional dependency (FD) $X \rightarrow Y$ means that for any two tuples t_1,t_2 in val(T):

if
$$\pi_X(t_1) = \pi_X(t_2)$$
 then $\pi_Y(t_1) = \pi_Y(t_2)$.

"if two tuples agree on their X-values, then they also agree on their Y-values"

Important:

FDs are determined by the semantics of the table. Not by any particular instance of a table!

sch(Book) = (ISBN, Title, Author)

What are the (interesting) functional dependencies?

```
sch(Book) = (ISBN, Title, Author)
```

What are the (interesting) functional dependencies?

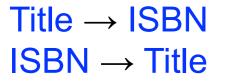
Scenario-1: we assume that Titles are unique, i.e., there cannot be two different ISBN's with the same title. Then:

```
Title → ISBN "Title gives me the ISBN" ISBN → Title "ISBN gives me the Title"
```

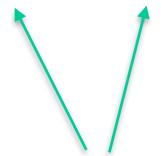
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What are the (interesting) functional dependencies?

Scenario-1: we assume that Titles are unique, i.e., there cannot be two different ISBN's with the same title. Then:



"Title gives me the ISBN" "ISBN gives me the Title"



these are sets of attributes, but we often omit the brackets '{' '}'.

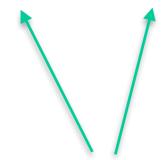
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What are the (interesting) functional dependencies?

Scenario-1: we assume that Titles are unique, i.e., there cannot be two different ISBN's with the same title. Then:

Title → ISBN ISBN → Title

"Title gives me the ISBN" "ISBN gives me the Title"



these are sets of attributes, but we often omit the brackets '{' '}'.

Question

What are the candidate keys of this table?

sch(Book) = (ISBN, Title, Author)

Scenario-2: Titles are **not** unique, there may be two different ISBNs with the **same title**.

What are now the (interesting) functional dependencies?

sch(Book) = (ISBN, Title, Author)

Scenario-2: Titles are **not** unique, there may be two different ISBNs with the **same title**.

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ISBN → Title — this one has not changed

any other one?

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Question

What are the candidate keys of this table?

sch(Teach) = (Course, Prof, Time)

Course	Prof		Time	9
cs101 cs101 cs311 cs477	Knuth Knuth Knuth Smith	İ	Fr, Th,	9-11 14-16 8-10 9-11

Important assumption: each course is taught by exactty one Prof.

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What are the (interesting) functional dependencies?

Does this table contain redundancy?

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Course	Prof	Time	Time	
	Knuth Knuth	1		
	Knuth			

Important assumption: each course is taught by exactty one Prof.

What are the (interesting) functional dependencies?

Does this table contain redundancy? — YES!

Superkeys and Keys

If X \to Y holds and $X \cup Y = \operatorname{Sch}(T)$ then X is a superkey of T. sch(T), but seen as a **set**

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A key (aka "candidate key") is a minimal superkey.

if we remove any element, then it ceases to be a superkey

There can be several keys (minimal superkeys). That is why we often say "candidate key" instead of "key."

Superkeys and Keys

sch(Teach) = (Course, Prof, Time)

Course	Prof		Time	
	Knuth Knuth	•		
	Knuth			

Important assumption: each course is taught by exactty one Prof.

What are the candidate keys of this table?

Course —> Prof

Prof, Time —> Course

Prof, Time Course, Time

Functional Dependencies ↔ Keys

Functional dependencies are a generalization of key constraints:

$$A_1, \ldots, A_n$$
 is a set of identifying attributes¹¹ ("superkey") in relation $R(A_1, \ldots, A_n, B_1, \ldots, B_m)$. \Leftrightarrow $A_1 \ldots A_n \to B_1 \ldots B_m$ holds.

Conversely, functional dependencies can be explained with keys.

$$A_1 \dots A_n \to B_1 \dots B_m$$
 holds for R . \Leftrightarrow

 A_1, \ldots, A_n is a set of identifying attributes in $\pi_{A_1, \ldots, A_n, B_1, \ldots, B_m}(R)$.

- → Functional dependencies are "partial keys".
- → A goal of this chapter is to turn FDs into **real keys**, because key constraints can easily be enforced by a DBMS.

¹¹If the set is also minimal, A_1, \ldots, A_n is a key (\nearrow slide 53).

What does it mean "interesting" functional dependency?

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also implies that

ISBN, Author → Title "ISBN and Author give me the Title" (in fact, we do not need the Author)

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ISBN, Author, Title → Title

ISBN, Author, Title → Title, Author

ISBN, Author, Title → Title, Author, ISBN

What does it mean "interesting" functional dependency?

```
 \begin{tabular}{ll} ISBN \to Title & "interesting" \\ ISBN, Author \to Title & "ISBN and Author give me the Title" & (in fact, we do not need the Author) \\ ISBN, Author, Title \to Title & "not so interesting" \\ ISBN, Author, Title \to Title, Author, ISBN & "not so interesting" \\ ISBN, Author, Title \to Title, Author, ISBN & "not so interesting" \\ \end{tabular}
```

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How many different functional dependencies exist for such a table T?

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- a.) 8
- b.) 27
- c.) 49
- d.) 64

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b.) 27
c.) 49
d.) 64?

$$(2^3 - 1) * (2^3 - 1) = 7*7 = 49$$

number of **non-empty** sets with at most 3 elements

A functional dependency $X \rightarrow Y$ is trivial, if Y is a subset of X.

How many non-trivial FDs are there for a table with 3 columns?

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A functional dependency $X \to Y$ is completely non-rivial, if $X \cap Y = \emptyset$.

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How many completely non-trivial FD for a table w. 3 columns? **12**

$$ab \rightarrow c$$
 $b \rightarrow a$
 $ac \rightarrow b$ $b \rightarrow c$
 $bc \rightarrow a$ $b \rightarrow ac$
 $a \rightarrow b$ $c \rightarrow b$
 $a \rightarrow c$ $c \rightarrow a$
 $a \rightarrow bc$ $c \rightarrow ab$

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Still: if $a \rightarrow b$ holds, then we **do not** care that also $ac \rightarrow b$ holds.

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$ab \rightarrow c$	$b \rightarrow a$	Still: if $a \rightarrow b$ holds, then we do not
$ac \rightarrow b$	$b \rightarrow c$	care that also $ac \rightarrow b$ holds.
$bc \rightarrow a$	$b \rightarrow ac$	
$a \rightarrow b$	$c \rightarrow p$	The FD $a \rightarrow b$ implies $ac \rightarrow b$
$a \rightarrow c$	$c \rightarrow a$	(and it is strictly smaller).
$a \rightarrow bc$	$c \rightarrow ab$	

A functional dependency with *m* attributes on the right-hand side

$$A_1 \dots A_n \to B_1 \dots B_m$$

is **equivalent** to the *m* functional dependencies

$$A_1 \dots A_n \rightarrow B_1$$
 \vdots
 $A_1 \dots A_n \rightarrow B_m$

Often, functional dependencies **imply** one another.

 \rightarrow We say that a set of FDs \mathcal{F} entails another FD f if the FDs in \mathcal{F} guarantee that f holds as well.

Reasoning over Functional Dependencies

Intuitively, we want to (re-)write relational schemas such that

- redundancy is minimized (and thus also update anomalies) and
- the system can still guarantee the same integrity constraints.

Functional dependencies allow us to **reason** over the latter.

Closure of a Set of Functional Dependencies

Given a set of functional dependencies \mathcal{F} , the set of all functional dependencies entailed by \mathcal{F} is called the **closure of** \mathcal{F} , denoted \mathcal{F}^+ : 12

$$\mathcal{F}^+:=\left\{\, lpha o eta \mid lpha o eta \,\, ext{entailed} \,\, ext{by} \,\, \mathcal{F} \,
ight\}$$
 .

Closures can be used to express **equivalence** of sets of FDs:

$$\mathcal{F}_1 \equiv \mathcal{F}_2 \Leftrightarrow \mathcal{F}_1^+ = \mathcal{F}_2^+$$
.

If there is a way to **compute** \mathcal{F}^+ for a given \mathcal{F} , we can test

- whether a given FD $\alpha \to \beta$ is entailed by \mathcal{F} ($\rightsquigarrow \alpha \to \beta \stackrel{?}{\in} \mathcal{F}^+$)
- whether two sets of FDs, \mathcal{F}_1 and \mathcal{F}_2 , are equivalent.

¹²Let α , β , ... denote sets of attributes.

6.) Armstrong Axioms

Armstrong Axioms

 \mathcal{F}^+ can be computed from \mathcal{F} by repeatedly applying the so-called **Armstrong axioms** to the FDs in \mathcal{F} :

■ **Reflexivity:** ("trivial functional dependencies")

If $\beta \subseteq \alpha$ then $\alpha \to \beta$.

Augmentation:

If
$$\alpha \to \beta$$
 then $\alpha \gamma \to \beta \gamma$.

Transitivity:

If
$$\alpha \to \beta$$
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It can be shown that the three Amstrong axioms are **sound** and **complete**: exactly the FDs in \mathcal{F}^+ can be generated from those in \mathcal{F} .

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Exercise: Show that

$$A_1 \dots A_n \to B_1 \dots B_m$$

is **equivalent** to the *m* functional dependencies

$$A_1 \dots A_n \rightarrow B_1$$

using the Armstrong Axioms.

$$\vdots \qquad \qquad \vdots \\ A_1 \dots A_n \quad \rightarrow \quad B_m$$

