Universität Bremen

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Mathe 2
Blatt P11 Musterlösung

Gruppe: Mathe 2 Tutor:innen

Für dieses Blatt sind die folgenden Regeln wichtig:

11.6 (Linearität):
$$\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

11.6 (Linearität):
$$\int (k \cdot f(x))\,dx = k \cdot \int f(x)\,dx$$

11.8 (Partielle Integration):
$$\int f(x)g'(x)\,dx = f(x) \cdot g(x) - \int f'(x)g(x)\,dx$$

11.10 (Integration durch Substitution):
$$\int f(g(x)) \cdot g'(x)\,dx = \int f(u)\,du$$

Präsenzübungen

P1 Bestimmen Sie die folgenden Integrale

a) $\int 15\sqrt{x\sqrt{x\sqrt{x}}}\,dx$

$$\begin{aligned}
\int 15\sqrt{x\sqrt{x\sqrt{x}}}\,dx &= \int 15\sqrt{x\sqrt{x \cdot x^{\frac{1}{2}}}}\,dx \\
&= \int 15\sqrt{x\sqrt{x^{\frac{3}{2}}}}\,dx \\
&= \int 15\sqrt{x \cdot x^{\frac{3}{4}}}\,dx \\
&= \int 15\sqrt{x^{\frac{7}{4}}}\,dx \\
&= \int 15x^{\frac{7}{8}}\,dx \\
&= 15\int x^{\frac{7}{8}}\,dx \\
&= 15 \cdot \frac{x^{\frac{15}{8}}}{\frac{15}{8}} \\
&= 8 \cdot x^{\frac{15}{8}}
\end{aligned}$$

b) $\int x^2 \sin(2x)\, dx$

$$\begin{aligned}
\int \overbrace{x^2}^{f(x)} \overbrace{\sin(2x)}^{g'(x)}\, dx &= -\frac{1}{2}\cos(2x)x^2 - \int -\frac{1}{2}\cos(2x)2x\, dx \\
&= -\frac{1}{2}\cos(2x)x^2 + \int \underbrace{\cos(2x)}_{g'(x)} \underbrace{x}_{f(x)}\, dx \\
&= -\frac{1}{2}\cos(2x)x^2 + \frac{1}{2}\sin(2x)x - \int \frac{1}{2}\sin(2x) \cdot 1\, dx \\
&= -\frac{1}{2}\cos(2x)x^2 + \frac{1}{2}\sin(2x)x - \frac{1}{2}\int \sin(2x)\, dx \\
&= -\frac{1}{2}\cos(2x)x^2 + \frac{1}{2}\sin(2x)x - \frac{1}{2}(-\frac{1}{2})\cos(2x) \\
&= \frac{1}{4}(-2\cos(2x)x^2 + 2\sin(2x)x + \cos(2x)) \\
&= \frac{1}{4}((1-2x^2)\cos(2x) + 2x\sin(2x))
\end{aligned}$$

Nebenrechnungen:

$$\begin{aligned}
\int \sin(2x)\, dx &= \frac{1}{2} \cdot \int \overbrace{2}^{g'(x)} \cdot \underbrace{\sin}_{f(x)} (\overbrace{2x}^{g(x)})\, dx \\
&= \frac{1}{2} \cdot \int \sin(u)\, du \\
&= \frac{1}{2} \cdot -\cos(u) = -\frac{1}{2}\cos(2x)
\end{aligned}$$

$$\begin{aligned}
\int \cos(2x)\, dx &= \frac{1}{2} \cdot \int \overbrace{2}^{g'(x)} \cdot \underbrace{\cos}_{f(x)} (\overbrace{2x}^{g(x)})\, dx \\
&= \frac{1}{2} \cdot \int \cos(u)\, du \\
&= \frac{1}{2} \cdot \sin(u) = \frac{1}{2}\sin(2x)
\end{aligned}$$

c) $\int 2x \sin(x^2)\, dx$

$$\begin{aligned}
\int \underbrace{2x}_{g'(x)} \overbrace{\sin}^{f(x)} (\underbrace{x^2}_{g(x)})\, dx &= \int \sin(u)\, du \\
&= -\cos(u) \\
&= -\cos(x^2)
\end{aligned}$$

d) $\int 2t \cdot (t^2-1)^4\, dt$

$$\begin{aligned}
\int \underbrace{2t}_{g'(x)} \cdot (\underbrace{t^2-1}_{g(x)})^{\overbrace{4}^{f(x)}}\, dt &= \int u^4\, du \\
&= \frac{u^5}{5} \\
&= \frac{1}{5}(t^2-1)^5
\end{aligned}$$

e) $\int \left(\frac{1}{y} + \exp(y)\right)\, dy$

$$\begin{aligned}\int \left(\frac{1}{y} + \exp(y)\right)\, dy &= \int \frac{1}{y}\, dy + \int \exp(y)\, dy \\ &= \ln|y| + \exp(y)\end{aligned}$$

f) $\int \cos(3x)\, dx$

$$\begin{aligned}\int \cos(3x)\, dx &= \int \frac{3}{3}\cos(3x)\, dx \\ &= \frac{1}{3}\int \underbrace{3}_{g'(x)} \overbrace{\cos}^{f(x)}(\underbrace{3x}_{g(x)})\, dx \\ &= \frac{1}{3}\int \cos(u)\, du \\ &= \frac{1}{3}\sin(u) \\ &= \frac{1}{3}\sin(3x)\end{aligned}$$

g) $\int x\sin(x^2+2)\, dx$

$$\begin{aligned}\int x\sin(x^2+2) &= \int \frac{2}{2} x\sin(x^2+2)\, dx \\ &= \frac{1}{2}\int \underbrace{2x}_{g'(x)} \overbrace{\sin}^{f(x)}(\underbrace{x^2+2}_{g(x)})\, dx \\ &= \frac{1}{2}\int \sin(u)\, du \\ &= \frac{1}{2}\cdot(-\cos(u)) \\ &= -\frac{1}{2}\cos(x^2+2)\end{aligned}$$

P2

Sei $f : [a,b] \to \mathbb{R} \setminus \{0\}$ stetig differenzierbar. Bestimmen Sie $\int \frac{f'(x)}{f(x)}\, dx$.

Sei $g(x) = \frac{1}{x}$, dann ist $f'(x)g(f(x)) = \frac{f'(x)}{f(x)}$. Also

$$\int \frac{f'(x)}{f(x)}\, dx = \int \frac{1}{u}\, du = \ln(|u|) = \ln(|f(x)|)$$