Universität Bremen

Prof. Dr. Anastasios Stefanou
Sommersemester 2023

Mathe 2

Blatt P12 Musterlösung

Gruppe: Mathe 2 Tutor:innen

Für dieses Blatt sind die folgenden Regeln wichtig:

11.6 (Linearität):
$$\int_b^a (f(x) + g(x))\, dx = \int_b^a f(x)\, dx + \int_b^a g(x)\, dx$$

11.6 (Linearität):
$$\int_b^a (k \cdot f(x))\, dx = k \cdot \int_b^a f(x)\, dx$$

11.8 (Partielle Integration):
$$\int_b^a f(x)g'(x)\, dx = [f(x) \cdot g(x)]_b^a - \int_b^a f'(x)g(x)\, dx$$

11.10 (Integration durch Substitution):
$$\int_b^a f(g(x)) \cdot g'(x)\, dx = \int_{g(b)}^{g(a)} f(u)\, du$$

Präsenzübungen

P1 Bestimmen Sie die folgenden Integrale

a) $\displaystyle\int \frac{1}{x \cdot \ln(2x)}\, dx$

$$\int \frac{1}{x \cdot \ln(2x)}\, dx = \int \underbrace{\frac{1}{\ln(2x)}}_{g(x)} \cdot \underbrace{\frac{2}{2x}}_{g'(x)}\, dx$$
$$= \int \frac{g'(x)}{g(x)}\, dx$$
$$= \ln(|g(x)|) = \boxed{\ln(|\ln(2x)|)}$$

b) $\displaystyle\int_0^2 2xe^{x^2}\, dx$

$$\int_0^2 \underbrace{2x}_{g'(x)}\, e^{\overbrace{x^2}^{g(x)}}\, dx = \int_{g(0)}^{g(2)} e^u\, du$$
$$= \int_0^4 e^u\, du$$
$$= e^4 - e^0 = \boxed{e^4 - 1}$$

c) $\displaystyle\int_e^{e^2} \frac{\ln(\ln(x))}{x \cdot \ln(x)}\, dx$

$$\int_e^{e^2} \frac{\ln(\ln(x))}{x \cdot \ln(x)}\, dx = \int_e^{e^2} \overbrace{\ln(\ln(x))}^{f(x)}\, \overbrace{\frac{1}{x \cdot \ln(x)}}^{f'(x)}\, dx$$
$$= \left[\frac{\ln(\ln(x)) \cdot \ln(\ln(x))}{2}\right]_e^{e^2}$$
$$= \left[\frac{\ln^2(\ln(x))}{2}\right]_e^{e^2}$$
$$= \frac{\ln^2(\ln(e^2))}{2} - \frac{\ln^2(\ln(e))}{2}$$
$$= \frac{\ln^2(2) - \ln^2(1)}{2} = \boxed{\frac{\ln^2(2)}{2}}$$

Nebenrechnung:

$$\int_a^b f(x)f'(x)\, dx = [f(x)f(x)]_a^b - \int_a^b f(x)f'(x)\, dx$$
$$2\int_a^b f(x)f'(x)\, dx = [f(x)f(x)]_a^b$$
$$\int_a^b f(x)f'(x)\, dx = \frac{1}{2}\, [f(x)f(x)]_a^b$$

d) $\displaystyle\int_0^1 e^x x\, dx$

$$\int_0^1 \underbrace{e^x}_{g'(x)}\, \overbrace{x}^{f(x)}\, dx = [e^x \cdot x]_0^1 - \int_0^1 e^x \cdot 1\, dx$$
$$= [e^x \cdot x]_0^1 - [e^x]_0^1$$
$$= [e^x(x-1)]_0^1$$
$$= e^1(1-1) - e^0(0-1)$$
$$= 0 - (-1) = \boxed{1}$$

e) $\int_0^{\frac{\pi}{4}} x^2 \sin(x)\, dx$

$$\int_0^{\frac{\pi}{4}} \underbrace{x^2}_{f(x)} \underbrace{\sin(x)}_{g'(x)}\, dx = \left[-x^2 \cos(x)\right]_0^{\frac{\pi}{4}}$$

$$+ \int_0^{\frac{\pi}{4}} \underbrace{2x}_{f(x)} \underbrace{\cos(x)}_{g'(x)}\, dx$$

$$= \left[-x^2 \cos(x)\right]_0^{\frac{\pi}{4}} + [2x \sin(x)]_0^{\frac{\pi}{4}}$$

$$- \int_0^{\frac{\pi}{4}} 2 \sin(x)\, dx$$

$$= \left[-x^2 \cos(x)\right]_0^{\frac{\pi}{4}} + [2x \sin(x)]_0^{\frac{\pi}{4}}$$

$$- [-2 \cos(x)]_0^{\frac{\pi}{4}}$$

$$= \left[2x \sin(x) - x^2 \cos(x) + 2 \cos(x)\right]_0^{\frac{\pi}{4}}$$

$$= \left[2x \sin(x) - (2 - x^2) \cos(x)\right]_0^{\frac{\pi}{4}}$$

$$= 2\left(\frac{\pi}{4}\right) \sin\left(\frac{\pi}{4}\right) - \left(2 - \left(\frac{\pi}{4}\right)^2\right) \cos\left(\frac{\pi}{4}\right)$$

$$- 2 \cdot 0 \cdot \sin(0) - (2 - 0^2) \cos(0)$$

$$= \boxed{\left(-\sqrt{2} + \frac{\pi}{2\sqrt{2}} + \frac{\pi^2}{16\sqrt{2}}\right) + 2}$$

f) $\int_1^{e} x \ln(x)\, dx$

$$\int_1^e \underbrace{x}_{g'(x)} \overbrace{\ln(x)}^{f(x)}\, dx = \left[\frac{x^2}{2} \ln(x)\right]_1^e - \int_1^e \frac{x^2}{2} \frac{1}{x}\, dx$$

$$= \left[\frac{x^2}{2} \ln(x)\right]_1^e - \frac{1}{2} \int_1^e x\, dx$$

$$= \left[\frac{x^2}{2} \ln(x)\right]_1^e - \frac{1}{2} \left[\frac{x^2}{2}\right]_1^e$$

$$= \left[\frac{x^2}{2} \ln(x) - \frac{1}{2} \frac{x^2}{2}\right]_1^e$$

$$= \left[\frac{x^2}{2} \frac{2}{2} \ln(x) - \frac{1}{2} \frac{x^2}{2}\right]_1^e$$

$$= \left[\frac{x^2}{4} (2 \ln(x) - 1)\right]_1^e$$

$$= \left(\frac{e^2}{4} (2 \ln(e) - 1)\right)$$

$$- \left(\frac{1^2}{4} (2 \ln(1) - 1)\right)$$

$$= \left(\frac{e^2}{4} (1)\right)$$

$$- \left(\frac{1}{4} (-1)\right)$$

$$= \boxed{\frac{1}{4} \left(e^2 + 1\right)}$$