Exercises for the lecture

Mathematics III

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Solutions to Exercise 01

Tasks

1. **Modelling**

   (a) Let Ω_{ij} denote the number of goals scored by the i^{th} robot in the j^{th} game, for $i = 1, 2, \ldots, 5$ and $j = 1, 2, \ldots, 7$. Then, $\Omega_{ij} = \{\mathbb{N}\} = \{0, 1, 2, \ldots\}$. The sample space is a 5×7 matrix with entries Ω_{ij}. The result is countable.

   (b) Let $t_j(t \geq 0)$ denote the time to failure for the j^{th} gadget, for $j = 1, \ldots, n$. Then, $\Omega = \{t_1, t_2, \ldots, t_n\}$.The result is uncountable.

   (c) Let Ω denote the functionality for a laptop. Then, $\Omega = \{1, 2, 3, 4, 5\}$. The total sample space is the cartesian product $\Omega_r \times \Omega_s, \quad r, s = 1, \ldots, 25$ with elements $\omega_{r,s} \in \Omega$. This result is countable.

2. **Simple dice roll**

   (a) $A = \{2^0, 2^1, 2^2\} = \{1, 2, 4\}$.

   (b) $B = \{\Omega\}$.

   (c) $C = \{\phi\}$.

3. **Power set**

   (a) For each $A \subseteq \Omega$, a binary decision can be made by the function

   $$f_A(\omega) = \begin{cases} 1, & \omega \in A \\ 0, & \omega \notin A \end{cases}$$

   for all $\omega \in \Omega$, which can then be assigned in the binary string. Here, a bit represents exactly one element.

   (b) It is known that $\mathcal{P}(\Omega) = 2^{|\Omega|} = 2^m$, so there must be a bijection onto the variations (2^m) of the binary strings.

4. **Multiple select task**

   (a) FALSE. $2^N = 2^6 = 64$, the cardinality of the power set (which is the set of all subsets of a set) but each subset contains at most 6 elements.

   (b) TRUE. Compact set implies the set is closed and bounded. Since the interval $[a, b]$ is perfect (i.e., a set $S \subset \mathbb{R}$ is perfect if it is closed and every point of S is an accumulation point).

   (c) TRUE. $\mathcal{A}$ must be an algebra first (closed under finite unions) for it to be a sigma algebra (closed under countable union).

   (d) FALSE. Suppose Ω is the set of all natural numbers, let the set system be all finite subsets of it, then infinite union of these finite subsets yields the natural numbers which are no longer the set system.