University of Bremen

Institute for Statistics

Daniel Ochieng
MZH, Room 7240
E-Mail: dochieng@uni-bremen.de

Exercises for the lecture

Mathematics III

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Solutions to Exercise 03

Tasks

9. **Urn model.**

   (a) Using probability trees,
   $$P(RR) = \frac{r}{r+s} * \frac{r-1}{(r+s)-1}.$$

   (b)
   $$P(RS) = \frac{r}{r+s} * \frac{s}{(r+s)-1}.$$

10. **Bayes formula.**

    Let $P(G) = 10/12 = 0.83$ be the prob. that an image is original, $P(G^c) = 2/12 = 0.16$ be the prob. that an image is fake. Further let $P(F)$ be the prob. that the expert says an image is a forgery, and $P(F^c)$ the prob. that the expert says the image is an original. Then,
    $$\begin{aligned} P(F|G^c) &= 0.90, \\ P(F^c|G) &= 0.90, \\ P(F^c|G^c) &= 1 - P(F|G^c), \\ &= 1 - 0.90, \\ &= 0.1. \end{aligned}$$

    We need to find,
    $$\begin{aligned} P(G|F^c) &= \frac{P(F^c|G)P(G)}{P(F^c|G)P(G) + P(F^c|G^c)P(G^c)}, \\ &= \frac{(0.9 * 0.83)}{(0.9 * 0.83) + (0.16 * 0.1)}, \\ &= 0.9790. \end{aligned}$$

11. **Total probability.**

    Let $P(D) :=$ Prob. of an item being defective and $P(B_i) :=$ Prob. of an item coming from the i^{th} box, for $i = 1, 2, 3$.
    $$\begin{aligned} P(D) &= P(D \cap B_1) + P(D \cap B_2) + P(D \cap B_3), \\ &= P(D|B_1)P(B_1) + P(D|B_2)P(B_2) + P(D|B_3)P(B_3), \\ &= \left(\frac{5}{12} * \frac{1}{3}\right) + \left(\frac{3}{8} * \frac{1}{3}\right) + \left(\frac{2}{9} * \frac{1}{3}\right), \\ &= \frac{73}{216}, \\ &= 0.3379. \end{aligned}$$

12. **Multiple select task.**

(a) FALSE.

$$P(B|A) = \frac{P(A \cap B)}{P(A)},$$
$$P(A \cap B) = \frac{1}{2} * \frac{1}{4} \neq 0.$$

(b) TRUE. If $A \subset B$, then $P(A) \leq P(B)$,

$$P(A|B) = \frac{P(A \cap B)}{P(B)},$$
$$\therefore P(A) = \frac{1}{4} < P(B) = \frac{1}{2}.$$

(c) TRUE.

$$\begin{aligned} P(A^c|B^c) &= \frac{P(A^c \cap B^c}{P(B^c)}, \\ &= \frac{P(B) - P(A \cap B)}{1 - P(B)}, \ (A \subseteq B), \\ &= \frac{1/2 - 1/8}{1 - 1/2} = 3/4. \end{aligned}$$

(d) FALSE.

$$\begin{aligned} P(A|B^c) &= \frac{P(A \cap B^c)}{P(B^c}, \\ &= \frac{P(A) - P(A \cap B)}{1 - P(B)}, \\ &= \frac{1/4 - 1/8}{1 - 1/2}, \\ &= 1/4. \end{aligned}$$

Hence,

$$P(A|B) + P(A|B^c) = 1/2 \neq 1.$$